Unit 03

# LOGARITHMS

**Scientific Notation**

A number written in the form $a\times10^n$, where $1\le a<10$ and $n$ is an integer, is called the scientific notation.

**Example**

Write each of the following ordinary numbers in scientific notation

(i) 30600 (ii) 0.000058

**Solution**

(i) $30600 = 3.06\times10^4$

(move decimal point four places to the left)

(ii) $0.000058 = 5.8\times10^{-5}$

(move decimal point five places to the right)

**Example**

Change each of the following numbers from scientific notation to ordinary notation.

(i) $6.35\times10^6$ (ii) $7.61\times10^{-4}$

**Solution**

(i) $6.35\times10^6 = 6350000$

(move the decimal point six places to the right)

(ii) $7.61\times10^{-4} = 0.000761$

(move the decimal point four places to the left)

## Exercise 3.1

**Q1. Express each of the following numbers in scientific notation.**

i) 5700

Sol: $5700 = 5.7\times10^3$ (move decimal point three places to left)

ii) 49,800,000

Sol: $49{,}800{,}000 = 4.98\times10^7$ (move decimal point seven places to left)

iii) 96,000,000

Sol: $96{,}000{,}000 = 9.6\times10^7$ (move decimal point seven places to left)

iv) 416.9

Sol: $416.9 = 4.169\times10^2$ (move decimal point two places to left)

v) 83,000

Sol: $83{,}000 = 8.3\times10^4$ (move decimal point four places to left)

vi) 0.00643

Sol: $0.00643 = 6.43\times10^{-3}$ (move decimal point three places to right)

vii) 0.0074

Sol: $0.0074 = 7.4\times10^{-3}$ (move decimal point three places to right)

viii) 60,000,000

Sol: $60{,}000{,}000 = 6.0\times10^7$ (move decimal point seven places to left)

ix) 0.00000000395

Sol: $0.00000000395 = 3.95\times10^{-9}$ (move decimal point nine places to right)

x) $\dfrac{275,000}{0.0025}$

**Sol:** $\dfrac{275,000}{0.0025}$

$= \dfrac{2.75\times10^{5} \text{ (move decimal point five places to left)}}{2.5\times10^{-3} \text{ (move decimal point three places to right)}}$

**Q2. Express the following numbers in ordinary notation.**

i) $6\times10^{-4}$

**Sol:** $6\times10^{-4} = 0.0006$ (move decimal point four places to left)

ii) $5.06\times10^{10}$

**Sol:** $5.06\times10^{10} = 50,600,000,000$ (move decimal point ten places to right)

iii) $9.018\times10^{-6}$

**Sol:** $9.018\times10^{-6} = 0.000009018$ (move decimal point six places to left)

iv) $7.865\times10^{8}$

**Sol:** $7.865\times10^{8} = 786,500,000$ (move decimal point eight places to right)

## Logarithm of a Real Number

If $a^x = y$ then $x$ is called the logarithm of $y$ to the base '$a$' and is written as $\log_a y = x$, where $a > 0, a \neq 1$ and $y > 0$

i.e., the logarithm of a number y to the base '$a$' is the index $x$ of the power to which a must be raised to get that number y.

The relations $a^x = y$ and $\log_a y = x$ are equivalent. When one relation is given, it can be converted into the other. Thus

$$\boxed{a^x = y \Leftrightarrow \log_a y = x}$$

**Example**

Find $\log_4 2$, i.e., find log of 2 to the base 4.

**Solution**

Let $\log_4 2 = x$

Then its exponenetial form is $4^x = 2$

i.e., $2^{2x} = 2^1 \Rightarrow 2x = 1$

$\therefore \quad x = \dfrac{1}{2} \Rightarrow \log_4 2 = \dfrac{1}{2}$

**Deductions from Definition of Logarithm**

1. Since $a^0 = 1$, $\log_a 1 = 0$
2. Since $a^1 = a$, $\log_a a = 1$

**Common Logarithm or Brigg's Logarithm**

If the base of logarithm is taken as 10 then logarithm is called Common Logarithm.

**Characteristic**

The integral part of the logarithm of any number is called the characteristic.

**Characteristic of Logarithm of a number > 1**

The characteristic of the logarithm of a number greater than 1 is always one less than the number of digits in the integral part of the number.

When a number $b$ is written in the scientific notation, i.e., in the form $b = a \times 10^n$ where $1 \le a < 10$, the power of 10 i.e., $n$ will give the characteristic of $\log b$.

**Examples**

| Number | Scientific Notation | Characteristic of the Logarithm |
|---|---|---|
| 1.02 | $1.02 \times 10^0$ | 0 |
| 99.6 | $9.96 \times 10^1$ | 1 |
| 102 | $1.092 \times 10^2$ | 2 |
| 1662.4 | $1.6624 \times 10^3$ | 3 |

## Characteristic of Logarithm of a Number < 1

The characteristic of the logarithm of a number less than 1, is always negative and one more than the number of zeroes immediately after the decimal point of the number.

**Example**

Write the characteristic of the log of following numbers by expressing them in scientific notation and noting the power of 10.

0.872, 0.02, 0.00345

| Number | Scientific Notation | Characteristic of the Logarithm |
|---|---|---|
| 0.872 | $8.72 \times 10^{-1}$ | -1 |
| 0.02 | $2.0 \times 10^{-2}$ | -2 |
| 0.00345 | $3.45 \times 10^{-3}$ | -3 |

**Mantissa**

The fractional part of the logarithm of a number is called the mantissa. Mantissa is always positive

**Example**

Find the mantissa of the logarithm of 43.254

**Solution**

Rounding off 43.254 we consider only the four significant digits 4325.

(i) We first locate the row corresponding to 43 in the log tables and

(ii) Proceed horizontally till we reach the column corresponding to 2. The number at the intersection is 6355.

(iii) Again proceeding horizontally till the mean difference column corresponding to 5 intersects this row, we get the number 5 at the intersection.

(iv) Adding the two numbers 6355 and 5 we get .6360 as the mantissa of the logarithm of 43.25.

**Example**

Find the mantissa of the logarithm of 0.002347

**Solution**

Here also, we consider only the four significant digits 2347

We first locate the row corresponding to 23 in the logarithm tables and proceed as before.

Along the same row to its intersection with the column corresponding to 4 the resulting number is 3692. The number at the intersection of this row and the mean difference column corresponding to 7 is 13. Hence the sum of 3692 and 13 gives the mantissa of the logarithm of 0.0023476 as 0.3705

**Example**

Find (i) log 278.23

(ii) log 0.07058

**Solution**

(i) 278.23 can be rounded off as 278.2

The characteristic is 2 and the mantissa, using log tables, is .4443

$\therefore \quad \log 278.23 = 2.4443$

(ii) The characteristic of log 0.07058 is $-2$ which is written as $\bar{2}$ by convention.

Using log tables the mantissa is .8487, so that

Log 0.07058 = $\bar{2}.8487$

**Example**

Find the numbers whose logarithms are

(i) 1.3247 (ii) $\bar{2}.1324$

**Solution**

**(i) 1.3247**

Reading along the row corresponding to .32 (as mantissa = 0.3247), we get 2109 at the intersection of this row with the column corresponding to 4. The number at the intersection of this row and the mean difference column corresponding to 7 is 3. Adding 2109 and 3 we get 2112.

Since the characteristic is 1, it is increased by 1 (because there should be two digits in the integral part) and therefore the decimal point is fixed after two digits from left in 2112.

Hence antilog of 1.3247 is 21.12.

(ii) $\bar{2}$**.1324**

Proceeding as in (i) the significant figures corresponding to the mantissa 0.1324 are 1356. Since the characteristic is $\bar{2}$, its numerical value 2 is decreased by 1. Hence there will be one zero after the decimal point.

Hence antilog of $\bar{2}.1324$ is 0.01356.



## Exercise 3.2

**Q1. Find the common logarithm of the following numbers.**

**i) 232.92**

232.92 can be rounded off as 232.9

Characteristic = 2

Mantissa = .3672

Hence log 232.92 = 2.3672

**ii) 29.326**

29.326 can be rounded off as 29.33

Characteristic = 1

Mantissa = .4673

Hence log 29.326 = 1.4673

**iii) 0.00032**

Characteristic = $\bar{4}$

Mantissa = .5051

Hence log 0.0032 = $\bar{4}.5051$

**iv) 0.3206**

Characteristic = $\bar{1}$

Mantissa = .5060

Hence log 0.3206 = $\bar{1}.5060$

**Q2. If log 31.09 = 1.4926, find the values of following:**

**i) log 3.109**

**Sol:** log 3.109

Characteristic = 0

Mantissa = .4926

So log 3.109 = 0.4926

**ii) log 310.9**

**Sol:** log 310.9

Characteristic = 2

Mantissa = .4926

So log 310.9 = 2.4926

**iii) log 0.003109**

**Sol:** log 0.003109

Characteristic = $\bar{3}$

Mantissa = .4926

So log 0.003109 = $\bar{3}.4926$

**iv) log 0.3109**

**Sol:** log 0.3109

Characteristic = $\bar{1}$

Mantissa = .4926

So log 0.3109 = $\bar{1}.4926$

**Q3. Find the numbers whose common logarithms are:**

i) 3.5621

let the number be x

log x = 3.5621

Characteristic = 3

Mantissa = .5621

x = antilog 3.5621 = 3648

x = 3648

Hence 3648 is the required number

ii) $\bar{1}.7427$

Let the number be x

Log x = $\bar{1}.7427$

Characteristic = $\bar{1}$

Mantissa = .7427

x = antilog $\bar{1}.7427$ = 0.5530

x = 0.5530

Hence 0.5530 is the required number

**Q4. What replacement for the unknown in each of following will make the statement true?**

i) $\log_3 81 = L$

In exponential form

$$3^L = 81$$

$$3^L = 3^4$$

$\Rightarrow$ $\boxed{L = 4}$ Bases are equal so exponents are equal

ii) $\log_a 6 = 0.5$

In exponential form

$$a^{0.5} = 6$$

$$a^{\frac{1}{2}} = 6$$

Squaring both side

$$\left(a^{\frac{1}{2}}\right)^2 = (6)^2$$

$$\boxed{a = 36}$$

iii) $\log_5 n = 2$

In exponential form

$$5^2 = n$$

$\Rightarrow$ $\boxed{n = 25}$

iv) $10^P = 40$

In logarithmic form

$\text{Log}_{10} 40 = P$

or log 40 = P

Characteristic = 1

Mantissa = .6021

So, P = 1.6021

**Q5. Evaluate**

i) $\log_2 \frac{1}{128}$

Let $x = \text{Log}_2 \frac{1}{128}$

In exponential form

$$2^x = \frac{1}{128}$$

$$2^x = \frac{1}{2^7}$$

$$2^x = 2^{-7}$$

$\Rightarrow$ $\boxed{x = -7}$

ii) log 512 to the base $2\sqrt{2}$

**Sol:** $\log_{2\sqrt{2}} 512$

Let $x = \log_{2\sqrt{2}} 512$

In exponential form

$$\left(2\sqrt{2}\right)^x = 512$$

$$\left(2\times 2^{\frac{1}{2}}\right)^x = 2^9$$

$$\left(2^{1+\frac{1}{2}}\right)^x = 2^9$$

$$\left(2^{\frac{3}{2}}\right)^x = 2^9$$

$$2^{\frac{3}{2}x} = 2^9$$

$$\Rightarrow \quad \frac{3}{2}x = 9$$

$$x = {}^{3}\cancel{9}\times\frac{2}{\cancel{3}}$$

$$\boxed{x=6}$$

**Q6. Evaluate the value of 'x' from the following statements.**

**i)** $\log_2 x = 5$

In exponential form

$$2^5 = x$$

$$\Rightarrow \quad \boxed{x=32}$$

**ii)** $\log_{81} 9 = x$

In exponential form

$$81^x = 9$$

$$\left(9^2\right)^x = 9$$

$$9^{2x} = 9^1$$

$$\Rightarrow \quad 2x = 1$$

$$\text{or} \quad \boxed{x=\frac{1}{2}}$$

**iii)** $\log_{64} 8 = \frac{x}{2}$

In exponential form

$$(64)^{\frac{x}{2}} = 8$$

$$\left(8^2\right)^{\frac{x}{2}} = 8$$

$$8^{\cancel{2}\times\frac{x}{\cancel{2}}} = 8$$

$$8^x = 8^1$$

$$\Rightarrow \quad \boxed{x=1}$$

**iv)** $\log_x 64 = 2$

In exponential form

$$x^2 = 64$$

$$x^2 = 8^2$$

$$\Rightarrow \quad \boxed{x=8}$$

**v)** $\log_3 x = 4$

In exponential form

$$3^4 = x$$

$$\Rightarrow \quad \boxed{x=81}$$

## Laws of Logarithm

In this section we shall prove the laws of logarithm and then apply them to find products, quotients, powers and roots of numbers.

(i) $\log_a(mn) = \log_a m + \log_a n$

(ii) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

(iii) $\log_a m^n = n\log_a m$

(iv) $\log_a n = \log_b n \times \log_a b$

or $= \dfrac{\log_b n}{\log_b a}$

**(i) $\log_a(mn) = \log_a m + \log_a n$:**

**Proof**

Let $\log_a m = x$ *and* $\log_a n = y$

Writing in exponential form

$a^x = m \ and \ a^y = n$

$\therefore \quad a^x \times a^y = mn$

i.e., $\quad a^{x+y} = mn$

or $\log_a(mn) = x + y = \log_a m + \log_a n$

Hence $\quad \log_a(mn) = \log_a m + \log_a n$

**Note**

(i) $\log_a(mn) \neq \log_a m \times \log_a n$

(ii) $\log_a m + \log_a n \neq \log_a(m+n)$

(iii) $\log_a(mnp..) = \log_a m + \log_a n + \log_a p + ..$

The rule given above is useful in finding the product of two or more numbers using logarithms.

**Example**

Evaluate $291.3 \times 42.36$

**Solution**

Let $x = 291.3 \times 42.36$

Then $\log x = \log(291.3 \times 42.36)$

$= \log 291.3 + \log 42.36$

$(\log_a mn = \log_a m + \log_a n)$

$= 2.4643 + 1.6269 = 4.0912$

$x = \text{antilog } 4.0912 = 12340$

**Example**

Evaluate $0.2913 \times 0.004236$.

**Solution**

Let $y = 0.2913 \times 0.004236$

Then $\log y = \log 0.2913 + \log 0.004236$

$\log y = \bar{1}.4643 + \bar{3}.6269$

$\log y = \bar{3}.0912$

$y = \text{anti log } \bar{3}.0912$

$y = 0.001234$

**(ii) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$**

**Proof**

Let $\quad \log_a m = x \ and \ \log_a n = y$

So that $\quad a^x = m \ and \ a^y = n$

$\therefore \quad \frac{a^x}{a^y} = \frac{m}{n} \Rightarrow a^{x-y} = \frac{m}{n}$

i.e.,

$\log_a\left(\frac{m}{n}\right) = x - y = \log_a m - \log_a n$

Hence $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

**Note**

(i) $\log_a\left(\frac{m}{n}\right) \neq \frac{\log_a m}{\log_a n}$

(ii) $\log_a m - \log_a n \neq \log_a(m-n)$

(iii) $\log_a\left(\frac{1}{n}\right) = \log_a 1 - \log_a n = -\log_a n .....$

$(\because \log_a 1 = 0)$

**Example**

Evaluate $\frac{291.3}{42.36}$

**Solution**

Let $\quad x = \frac{291.3}{42.36}$ so that $\log x = \log \frac{291.3}{42.36}$

Then $\log x = \log 291.3 - \log 42.36, ....$

$(\log_a \frac{m}{n} = \log_a m - \log_a n)$

$\log x = 2.4643 - 1.6269 = 0.8374$

Thus $\quad x = \text{antilog } 0.8374 = 6.877$

**Example**

Evaluate $\frac{0.0002913}{0.04236}$

**Solution**

Let $\quad y = \frac{0.0002913}{0.04236}$ so that

$\log y = \log\left(\frac{0.002913}{0.04236}\right)$

then $\quad \log y = \log 0.002913 - \log 0.04236$

$\log y \;=\; \bar{3}.4643 - \bar{2}.6269$

$= \bar{3} + (0.4643 - 0.6269) - \bar{2}$

$= \bar{3} - 0.1626 - \bar{2}$

$= \bar{3} + (1 - 0.1626) - 1 - \bar{2}$, (adding and subtracting 1)

$= \bar{2}.8374$

$[\because\ \bar{3} - 1 - \bar{2} = -3 - 1 - (-2) = -2 = \bar{2}]$

Therefore, $y = \text{antilog}\ \bar{2}.8374$

$y = 0.06877$

**(iii) $\log_a(m^n) = n\log_a m$:**

**Proof**

Let $\log_a m^n = x$, i.e., $a^x = m^n$

and $\log_a m = y$, i.e., $a^y = m$

Then $a^x = m^n = (a^y)^n$

i.e., $a^x = (a^y)^n = a^{yn} \Rightarrow x = ny$

i.e., $\log_a m^n = n\log_a m$

**Example**

Evaluate $\sqrt[4]{(0.0163)^3}$

**Solution**

Let $y = \sqrt[4]{(0.0163)^3} = (0.0163)^{3/4}$

$\log y = \frac{3}{4}(\log 0.0163)$

$= \frac{3}{4} \times \bar{2}.2122$

$= \frac{\bar{6}.6366}{4}$

$= \frac{\bar{8} + 2.6366}{4}$

$= \bar{2} + 0.6592 = \bar{2}.6592$

Hence $y = \text{antilog}\ \bar{2}.6592$

$= 0.04562$

**(iv) Change of Base Formula:**

$$\log_a n = \log_b n \times \log_a b \text{ or } \frac{\log_b n}{\log_b a}$$

**Proof**

Let $\log_b n = x$ so that $n = b^x$

Taking log to the base $a$, we have

$$\log_a n = \log_a b^x = x\log_a b = \log_b n \log_a b$$

Thus $\log_a n = \log_b n \log_a b$ ......(i)

Putting $n = a$ in the above result, we get

$\log_b a \times \log_a b = \log_a a = 1$

or $\log_a b = \frac{1}{\log_b a}$

Hence equation (i) gives

$$\log_a n = \frac{\log_b n}{\log_b a} \quad \text{......(ii)}$$

Using the above rule, a natural logarithm can be converted to a common logarithm and vice versa.

$$\log_e n = \log_{10} n \times \log_e 10 \text{ or } \frac{\log_{10} n}{\log_{10} e}$$

$$\log_{10} n = \log_e n \times \log_{10} e \text{ or } \frac{\log_e n}{\log_e 10}$$

The values of $\log_e 10$ and $\log_{10} e$ are available from the tables:

$$\log_e 10 = \frac{1}{0.4343} = 2.3026 \quad and$$

$$\log_{10} e = \log 2.718 = 0.4343$$

**Example**

Calculate $\log_2 3 \times \log_3 8$

**Solution**

We know that

$$\log_a n = \frac{\log_b n}{\log_b a}$$

$$\therefore \quad \log_2 3 \times \log_3 8 = \frac{\log 3}{\log 2} \times \frac{\log 8}{\log 3}$$

$$= \frac{\log 8}{\log 2} = \frac{\log 2^3}{\log 2}$$

$$= \frac{3\log 2}{\log 2} = 3$$

## Exercise 3.3

**Q1. Write the following into sum or difference.**

**i)** $\log(A \times B)$

**Sol:** $\log(A \times B) = \log A + \log B$

**ii)** $\log\frac{15.2}{30.5}$

**Sol:** $\log\frac{15.2}{30.5} = \log 15.2 - \log 30.5$

**iii)** $\log\frac{21 \times 5}{8}$

**Sol:** $\log\frac{21 \times 5}{8} = \log 21 + \log 5 - \log 8$

**iv)** $\log\sqrt[3]{\frac{7}{15}}$

**Sol:** $\log\sqrt[3]{\frac{7}{15}} = \log\left(\frac{7}{15}\right)^{\frac{1}{3}} = \frac{1}{3}\log\left(\frac{7}{15}\right)$

$$= \frac{1}{3}(\log 7 - \log 15)$$

**v)** $\log\frac{(22)^{\frac{1}{3}}}{5^3}$

**Sol:** $\log\frac{(22)^{\frac{1}{3}}}{5^3} = \log(22)^{\frac{1}{3}} - \log 5^3$

$$= \frac{1}{3}\log 22 - 3\log 5$$

**vi)** $\log\frac{25 \times 47}{29}$

$$= \log 25 + \log 47 - \log 29$$

**Q2. Express** $\log x - 2\log x + 3\log(x+1) - \log(x^2-1)$ **as a single logarithm**

**Sol:**

$$\log x - 2\log x + 3\log(x+1) - \log(x^2-1)$$

$$= \log x - \log x^2 + \log(x+1)^3 - \log(x^2-1)$$

$$= \log x + \log(x+1)^3 - \log x^2 - \log(x^2-1)$$

$$= \log\frac{x(x+1)^3}{x^2(x^2-1)}$$

$$= \log\frac{(x+1)^3}{x(x-1)(x+1)}$$

$$= \log\frac{(x+1)^2}{x(x-1)}$$

**Q3. Write the following in the form of a single logarithm.**

**i)** $\log 21 + \log 5$

**Sol:** $\log 21 + \log 5$

$$= \log 21 \times 5$$

**ii)** $\log 25 - 2\log 3$

$$= \log 25 - \log 3^2$$

$$= \log\frac{25}{3^2} = \log\frac{25}{9}$$

**iii)** $2\log x - 3\log y$

**Sol:** $2\log x - 3\log y$

$$= \log x^2 - \log y^3$$

$$= \log\frac{x^2}{y^3}$$

**iv)** $\log 5 + \log 6 - \log 2$

**Sol:** $\log 5 + \log 6 - \log 2$

$$= \log\frac{5 \times 6}{2}$$

**Q4. Calculate the following:**

i) $\log_3 2\times\log_2 81$

**Sol:** As we know that $\log_a n=\dfrac{\log_b n}{\log_b a}$

$$\therefore \log_3 2\times\log_2 81=\frac{\log 2}{\log 3}\times\frac{\log 81}{\log 2}$$

$$=\frac{\log 81}{\log 3}$$

$$=\frac{\log 3^4}{\log 3}$$

$$=\frac{4\log 3}{\log 3}$$

$$=4$$

ii) $\log_5 3\times\log_3 25$

Sol: As we know that

$$\log_a n=\frac{\log_b n}{\log_b a}$$

$$\log_5 3\times\log_3 25=\frac{\cancel{\log 3}}{\log 5}\times\frac{\log 25}{\log 3}$$

$$=\frac{\log 25}{\log 5}$$

$$=\frac{\log 5^2}{\log 5}$$

$$=\frac{2\cancel{\log 5}}{\cancel{\log 5}}$$

$$=2$$

**Q5. If** $\log 2=0.3010$, $\log 3=0.4771$, $\log 5=0.6990$**, then find the values of the following.**

i) $\log 32$

**Sol:** $\log 32$

$$=\log 2^5$$

$$=5\log 2$$

$$=5(0.3010)$$

$$=1.5050$$

ii) $\log 24$

$$=\log 8\times 3$$

$$=\log 2^3\times 3$$

$$=\log 2^3+\log 3$$

$$=3\log 2+\log 3$$

$$=3(0.3010)+0.4771$$

$$=0.9030+0.4771$$

$$=1.3801$$

iii) $\log\sqrt{3\frac{1}{3}}$

$$=\log\sqrt{\frac{10}{3}}$$

$$=\log\left(\frac{2\times5}{3}\right)^{\frac{1}{2}}$$

$$=\frac{1}{2}\log\left(\frac{2\times5}{3}\right)=\frac{1}{2}(\log 2+\log 5-\log 3)$$

$$=\frac{1}{2}(0.3010+0.6990-0.4771)$$

$$=\frac{1}{2}(0.5229)$$

$$=0.2615$$

iv) $\log\frac{8}{3}$

$$=\log\frac{2^3}{3}$$

$$=\log 2^3-\log 3$$

$$=3\log 2-\log 3$$

$$=3(0.3010)-0.4771$$

$$=0.4259$$

v) $\log 30$

$$=\log 2\times3\times5$$

$$=\log 2+\log 3+\log 5$$

$$=0.3010+0.4771+0.6990$$

$$=1.4771$$

## Applications of logarithm

**Example**

Show that

$$7\log\frac{16}{15}+5\log\frac{25}{24}+\log\frac{81}{80}=\log 2.$$

**Solution**

L.H.S $=7\log\frac{16}{15}+5\log\frac{25}{24}+\log\frac{81}{80}$

$=\quad 7[\log 16-\log 15]+5[\log 25-\log 24]+3[\log 81-\log 80]$

$=\quad 7[\log 2^4-\log(3\times 5)]+5[\log 5^2-\log(2^3\times 3)]+3[\log 3^4-\log(2^4\times 5)]$

$=7[4\log 2-\log 3-\log 5]+5[2\log 5-3\log 2-\log 3]+3[4\log 3-4\log 2-\log 5]$

$=\quad (28-15-12)\log 2+(-7-5+12)\log 3+(-7+10-3)\log 5$

$=\quad \log 2+0+0=\log 2\ =$ R.H.S

**Example**

Evaluate:

$$\sqrt[3]{\frac{0.07921\times(18.99)^2}{(5.79)^4\times 0.9474}}$$

Let y $=$

$$\sqrt[3]{\frac{0.07921\times(18.99)^2}{(5.79)^4\times 0.9474}}=$$

$$\left(\frac{0.07921\times(18.99)^2}{(5.79)^4\times 0.9474}\right)^{1/3}$$

Log y $=$

$$\frac{1}{3}\log\left(\frac{0.07921\times(18.99)^2}{(5.79)^4\times 0.9474}\right)$$

$=\frac{1}{3}[\log\{0.07921\times(18.99)^2\}-\log\{(5.79)^4\times 0.9474\}]$

$=\frac{1}{3}[\log 0.07921+2\log 18.99-4\log 5.79-\log 0.9474\}$

$=\ \frac{1}{3}[\bar{2}.8988+2(1.2786)-4(0.7627)-\bar{1}.9765]$

$=\ \frac{1}{3}[\bar{2}.8988+2.5572-3.0508-\bar{1}.9765]$

$=\frac{1}{3}[-2+0.8988+2.5572-3.0508+1-0.9765]$

$=\frac{1}{3}(\bar{2}.4287)$

$=\quad \frac{1}{3}(\bar{3}+1.4287)$

$=\quad \bar{1}+0.4762=\bar{1}.4762$

y $=$ antilog $\bar{1}.4762=0.2993$

**Example**

Given $A = A_0e^{-kd}$. If k = 2, what should be the value of d to make $A=\frac{A_o}{2}$?

**Solution**

Given that $A = A_0e^{-kd}$ $\Rightarrow$

$$\frac{A}{A_o}=e^{-kd}$$

Substituting k = 2 and $A=\frac{A_o}{2}$,

we get $\frac{1}{2}=e^{-2d}$

Taking common log on both sides,

$\log_{10}1-\log_{10}2=-2d\log_{10}e$,

where $e = 2.718$

$0-0.3010=-2d\ (0.4343)$

$$d\quad=\quad\frac{0.3010}{2\times 0.4343}=0.3465$$

## Exercise 3.4

**Q1. Use log tables to find the values of**

**i)** $0.8176 \times 13.64$

**Sol:** Let $x = 0.8176 \times 13.64$

Taking log of both sides

$\log x = \log\ 0.8176 \times 13.64$

$\log x = \log 0.8176 + \log 13.64$

$= \bar{1}.9125 + 1.1348$

$= -1 + 0.9125 + 1.1348$

$\log x = 1.0473$

| | | |
|---|---|---|
| Characteristics | = | 1 |
| Mantissa | = | .0473 |
| $x$ = antilog 1.0473 | = | 11.15 |

**ii)** $(789.5)^{\frac{1}{8}}$

**Sol:** Let $x = (789.5)^{\frac{1}{8}}$

Taking log of both sides

$\log x = \log(789.5)^{\frac{1}{8}}$

$= \frac{1}{8}\log(789.5)$

$= \frac{1}{8}(2.8974)$

$\log x = 0.3622$

| | | |
|---|---|---|
| Characteristics | = | 0 |
| Mantissa | = | .3622 |
| $x$ = antilog 0.3622 | = | 2.302 |

**iii)** $\dfrac{0.678 \times 9.01}{0.0234}$

Let $x = \dfrac{0.678 \times 9.01}{0.0234}$

Taking log of both sides

$\log x = \log \dfrac{0.678 \times 9.01}{0.0234}$

$= \log 0.678 + \log 9.01 - \log 0.0234$

$= \bar{1}.8312 + 0.9547 - (\bar{2}.3692)$

$= -1 + 0.8312 + 0.9547 - (-2 + 0.3692)$

$= -1 + 0.8312 + 0.9547 + 2 - 0.3692$

$\log x = 2.4167$

| | | |
|---|---|---|
| Characteristics | = | 2 |
| Mantissa | = | .4167 |
| $x$ = antilog 2.4167 | = | 261.0 |

**iv)** $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$

**Sol:** Let $x = \sqrt[5]{2.709} \times \sqrt[7]{1.239}$

Taking log of both sides

$\log x = \log(2.709)^{\frac{1}{5}} \times (1.239)^{\frac{1}{7}}$

$= \log(2.709)^{\frac{1}{5}} + \log(1.239)^{\frac{1}{7}}$

$= \frac{1}{5}\log(2.709) + \frac{1}{7}\log(1.239)$

$= \frac{1}{5}(0.4328) + \frac{1}{7}(0.0931)$

$= 0.0866 + 0.0133$

$\log x = 0.0999$

Characteristics = 0

Mantissa = .0999

$x =$ antilog 0.0999

$x = 1.259$

**v)** $\dfrac{(1.23)(0.6975)}{(0.0075)(1278)}$

**Sol:** Let $x = \dfrac{(1.23)(0.6975)}{(0.0075)(1278)}$

Taking log of both sides

$\log x = \log \dfrac{(1.23)(0.6975)}{(0.0075)(1278)}$

$= \log 1.23 + \log 0.6975 - \log 0.0075 - \log 1278$

$= 0.0899 + \bar{1}.8435 - \bar{3}.8751 - 3.1065$

$= 0.0899 - 1 + 0.8435 + 3 - 0.8751 - 3.1065$

$$\log x = -1.0482$$
$$= -2+2-1.0482$$
$$= -2+0.9518$$
$$\log x = \bar{2}.9518$$

Characteristics = $\bar{2}$

Mantissa = .9518

$x =$ antilog $\bar{2}.9518$ = 0.0895

vi) $\sqrt[3]{\dfrac{0.7214\times 20.37}{60.8}}$

Let $x = \sqrt[3]{\dfrac{0.7214\times 20.37}{60.8}}$

$$x = \left(\frac{0.7214\times 20.37}{60.8}\right)^{\frac{1}{3}}$$

Taking log of both sides

$$\log x = \log\left(\frac{0.7214\times 20.37}{60.8}\right)^{\frac{1}{3}}$$
$$= \frac{1}{3}\log\left(\frac{0.7214\times 20.37}{60.8}\right)$$
$$= \frac{1}{3}(\log 0.7214 + \log 20.37 - \log 60.8)$$
$$= \frac{1}{3}(\bar{1}.8582 + 1.3090 - 1.7839)$$
$$= \frac{1}{3}(-1+0.8582+1.3090-1.7839)$$
$$= \frac{1}{3}(-0.6167)$$
$$\log x = -0.2056$$
$$= -1+1-0.2056$$
$$= -1+0.7944$$
$$\log x = \bar{1}.7944$$

Characteristics = $\bar{1}$

Mantissa = .7944

$x =$ antilog $\bar{1}.7944$

$$= 0.6229$$

vii) $\dfrac{83\times\sqrt[3]{92}}{127\times\sqrt[5]{246}}$

**Sol:** Let $x = \dfrac{83\times\sqrt[3]{92}}{127\times\sqrt[5]{246}}$

$$x = \frac{83\times(92)^{\frac{1}{3}}}{127\times(246)^{\frac{1}{5}}}$$

Taking log of both sides

$$\log x = \log\frac{83\times(92)^{\frac{1}{3}}}{127\times(246)^{\frac{1}{5}}}$$
$$= \log 83 + \log(92)^{\frac{1}{3}} - \log 127 - \log(246)^{\frac{1}{5}}$$
$$= \log 83 + \frac{1}{3}\log(92) - \log 127 - \frac{1}{5}\log(246)$$
$$= 1.9191 + \frac{1}{3}(1.9638) - 2.1038 - \frac{1}{5}(2.391)$$
$$= 1.9191 + 0.6546 - 2.1038 - 0.4782$$
$$\log x = -0.0083$$
$$= -1+1-0.0083$$
$$= -1+0.9917$$
$$\log x = \bar{1}.9917$$

Characteristics = $\bar{1}$

Mantissa = .9917

$x =$ antilog $\bar{1}.9917$ = 0.9811

viii) $\dfrac{(438)^3\sqrt{0.056}}{(388)^4}$

**Sol:** Let $x = \dfrac{(438)^3\sqrt{0.056}}{(388)^4}$

$$x = \frac{(438)^3\times(0.056)^{\frac{1}{2}}}{(388)^4}$$

Taking log of both sides

$$\log x = \log \frac{(438)^3 \times (0.056)^{\frac{1}{2}}}{(388)^4}$$

$$= \log(438)^3 + \log(0.056)^{\frac{1}{2}} - \log(388)^4$$

$$= 3\log(438) + \frac{1}{2}\log(0.056) - 4\log(388)$$

$$= 3(2.6415) + \frac{1}{2}(\bar{2}.7482) - 4(2.5888)$$

$$= 3(2.6415) + \frac{1}{2}(-2 + 0.7482) - 4(2.5888)$$

$$= 7.9245 + \frac{1}{2}(-1.2518) - 10.3552$$

$$= 7.9245 - 0.6259 - 10.3552$$

$$\log x = -3.0566$$

$$= -4 + 4 - 3.0566$$

$$= -4 + 0.9434$$

$$\log x = \bar{4}.9434$$

Characteristic = $\bar{4}$

Mantissa = .9434

$x =$ antilog $\bar{4}.9434$ = 0.0008778

**Q2. A gas is expanding according to the law $PV^n = C$. Find C when P=80, V=3.1 and $n = \frac{5}{4}$.**

**Sol:** $PV^n = C$

Taking log of both sides:

$$\log PV^n = \log C$$

$$\log P + \log V^n = \log C$$

$$\log C = \log P + n \log V$$

Putting $P = 80$, $V = 3.1$ and $n = \frac{5}{4}$

$$\log C = \log 80 + \frac{5}{4}\log 3.1$$

$$= 1.9031 + \frac{5}{4}(0.4914)$$

$$= 1.9031 + 0.6143$$

$$\log C = 2.5174$$

Characteristic = 2

Mantissa = .5174

C = antilog 2.5174

C = 329.2 unit

**Q3. The formula $p = 90(5)^{-\frac{q}{10}}$ applies to the demand of a product, where 'q' is the number of units and p is the price of one unit. How many units will be demanded if the price is Rs. 18.00?**

**Sol:** $p = 90(5)^{-\frac{q}{10}}$

$q = ?$ and $p = Rs.\ 18.00$

As $p = 90(5)^{-\frac{q}{10}}$

$$18 = 90(5)^{\frac{-q}{10}}$$

Taking log of both sides

$$\log 18 = \log 90(5)^{-\frac{q}{10}}$$

$$\log 18 = \log 90 + \log(5)^{-\frac{q}{10}}$$

$$\log 18 - \log 90 = \frac{-q}{10}\log 5$$

$$10(\log 18 - \log 90) = -q \log 5$$

$$10(1.2553 - 1.9542) = -q(0.6990)$$

$$-6.989 = -q(0.6990)$$

$\Rightarrow$ $q(0.6990) = 6.989$

$$q = \frac{6.989}{0.6990}$$

$$q = 9.998$$

$q = 10$ approximately

So 10 units will be demanded

**OR**

$$p = 90\ (5)^{-q/10}$$

**Taking log of both sides**

$$\log p = \log 90\ (5)^{-q/10}$$

$$\log p = \log 90 + \log\ (5)^{-q/10}$$

$$\log p = \log 90 - \frac{q}{10}\log 5$$

$$\frac{q}{10}\log 5 = \log 90 - \log p$$

$$\frac{q}{10}\log 5 = \log 90 - \log 18$$

$$\frac{q}{10}\log 5 = \log\ \frac{90}{18}$$

$$\frac{q}{10}\log 5 = \log\ 5$$

$$\frac{q}{10} = \frac{\log 5}{\log 5}$$

$$\frac{q}{10} = 1$$

q = 10 Units

**Q4.** **If** $A = \pi r^2$

$$\pi = \frac{22}{7},\ r = 15,\ A = ?$$

As $A = \pi r^2$

Taking log of both sides

$$\log A = \log \pi r^2$$

$$= \log \pi + \log r^2$$

$$= \log \pi + 2\log r$$

$$= \log\frac{22}{7} + 2\log 15$$

$$= \log 22 - \log 7 + 2\log 15$$

$$= 1.3424 - 0.8451 + 2(1.1761)$$

$$= 1.3424 - 0.8451 + 2.3522$$

$$\log A = 2.8495$$

Characteristics = 2

Mantissa = .8495

A = antilog 2.8495

A = 707.1

**Q5.** **If** $v = \frac{1}{3}\pi r^2 h$, **find** $v$ **when** $\pi = \frac{22}{7}, r = 2.5$ **and** $h = 4.2$

**Sol:** $v = \frac{1}{3}\pi r^2 h$

$\pi = \frac{22}{7}$, $r = 2.5$ and $h = 4.2$, $v = ?$

As $v = \frac{1}{3}\pi r^2 h$

Taking log of both sides

$$\log v = \log\frac{1}{3}\pi r^2 h$$

$$= \log\frac{1}{3} + \log \pi + \log r^2 + \log h$$

$$= \log\frac{1}{3} + \log\frac{22}{7} + 2\log r + \log h$$

$$= \log 1 - \log 3 + \log 22 - \log 7 + 2\log 2.5 + \log 4.2$$

$$= 0 - 0.4771 + 1.3424 - 0.8451 + 2(0.3979) + 0.6232$$

$$\log v = 1.4392$$

Characteristics = 1

Mantissa = .4392

$v =$ antilog 1.4392

$v =$ 27.49

## Review Exercise 3

**Q3. Find the value of 'x' in the following.**

**i) $\log_3 x = 5$**

Sol. $\log_3 x = 5$

In exponential form

$x = 3^5$

$\Rightarrow$ $x = 243$

**ii) $\log_4 256 = x$**

Sol. $\log_4 256 = x$

In exponential form

$4^x = 256$

$4^x = 4^4$

$\Rightarrow$ $x = 4$

**iii) $\log_{625} 5 = \frac{1}{4}x$**

Sol. $\log_{625} 5 = \frac{1}{4}x$

In exponential form

$(625)^{\frac{1}{4}x} = 5$

$(5^4)^{\frac{1}{4}x} = 5$

$5^{4\times\frac{1}{4}x} = 5$

$5^x = 5^1$

$\Rightarrow$ $x = 1$

**iv) $\log_{64} x = -\frac{2}{3}$**

Sol. $\log_{64} x = -\frac{2}{3}$

In exponential form

$x = 64^{\frac{-2}{3}}$

$x = (4^3)^{\frac{-2}{3}}$

$= 4^{\cancel{3}\left(-\frac{2}{\cancel{3}}\right)}$

$x = 4^{-2}$

$x = \frac{1}{4^2}$

$x = \frac{1}{16}$

**Q4. Find the value of 'x' in the following.**

**i) $\log x = 2.4543$**

Characteristic = 2

Mantissa = .4543

x = antilog 2.4543

= 284.6

**ii) $\log x = 0.1821$**

Characteristic = 0

Mantissa = .1821

x = antilog 0.1821

= 1.521

**iii) $\log x = 0.0044$**

Characteristic = 0

Mantissa = .0044

x = antilog 0.0044

x = 1.010

**iv) $\log x = \bar{1}.6238$**

Characteristic = $\bar{1}$

Mantissa = .6238

x = antilog $\bar{1}.6238$

x = 0.4205

**Q5. If log2 = 0.3010, log3 = 0.4771 and log 5 = 0.6990, then find the values of the following.**

**i) log45**

**Sol.** log45

$= \log 3^2 \times 5$
$= \log 3^2 + \log 5$
$= 2\log 3 + \log 5$
$= 2(0.4771) + 0.6990$
$= 0.9542 + 0.6990$
$= 1.6532$

ii) $\log \frac{16}{15}$

$= \log \frac{2^4}{3\times 5}$
$= \log 2^4 - \log 3 - \log 5$
$= 4\log 2 - \log 3 - \log 5$
$= 4(0.3010) - 0.4771 - 0.6990$
$= 1.2040 - 0.4771 - 0.6990$
$= 0.0279$

iii) **log 0.048**

$= \log \frac{48}{1000}$
$= \log \frac{16\times 3}{10^3}$
$= \log \frac{2^4 \times 3}{2^3 \times 5^3}$
$= \log \frac{2\times 3}{5^3}$
$= \log 2 + \log 3 - \log 5^3$
$= \log 2 + \log 3 - 3\log 5$
$= 0.3010 + 0.4771 - 3(0.6990)$
$= -1.3189$
$= -2 + 2 - 1.3189$
$= -2 + 0.6811$
$= \bar{2}.6811$

**Q6. Simplify the following:**

i) $\sqrt[3]{25.47}$

Sol. Let $x = (25.47)^{\frac{1}{3}}$
Taking log of both sides
$\log x = \log (25.47)^{\frac{1}{3}}$
$= \frac{1}{3}\log(25.47)$
$= \frac{1}{3}(1.4060)$
$\log x = 0.4687$
Characteristic = 0
Mantissa = .4687
x = antilog 0.4687
x = 2.942

ii) $\sqrt[5]{342.2}$

Sol. Let $x = (342.2)^{\frac{1}{5}}$
Taking log of both sides
$\text{Log } x = \log (342.2)^{\frac{1}{5}}$
$= \frac{1}{5}\log(342.2)$
$= \frac{1}{5}(2.5343)$
$\log x = 0.5069$
Characteristic = 0
Mantissa = .5069
x = antilog 0.5069
x = 3.213

iii) $\frac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$

Sol: Let $x = \frac{(8.97)^3 \times (3.95)^2}{(15.37)^{\frac{1}{3}}}$
Taking log of both sides
$\log x = \log \frac{(8.97)^3 \times (3.95)^2}{(15.37)^{\frac{1}{3}}}$

$= \log(8.97)^3 + \log(3.95)^2 - \log(15.37)^{\frac{1}{3}}$

$= 3\log(8.97) + 2\log(3.95) - \frac{1}{3}\log(15.37)$

$= 3(0.9528) + 2(0.5966) - \frac{1}{3}(1.1867)$

$= 2.8584 + 1.1932 - 0.3956$

$\log x = 3.6560$

Characteristic = 3

Mantissa = .6560

x = antilog 3.6560

x = 4529

## Objective

**1.** If $a^x = n$, then ______

(a) $a = \log_x n$ (b) $x = \log_n a$

(c) $x = \log_a n$ (d) $a = \log_n x$

**2.** The relation of $y = \log_z x$ implies

(a) $x^y = z$ (b) $z^y = x$

(c) $x^z = y$ (d) $y^z = x$

**3.** The logarithm of unity to any base is ___

(a) 1 (b) 10

(c) e (d) 0

**4.** The logarithm of any number to itself as base is___

(a) 1 (b) 0

(c) −1 (d) 10

**5.** log e = ____ where e ≈ 2. 718

(a) 0 (b) 0.4343

(c) ∞ (d) 1

**6.** The value of $\log\left(\frac{p}{q}\right)$ is ___

(a) log p −log q

(b) $\frac{\log p}{\log q}$

(c) log p + log q

(d) log q − log p

**7.** $\log m^n$ can be written as

(a) $(\log m)^n$ (b) m log n

(c) n log m (d) log (mn)

**8.** $\log_b a \times \log_c b$ can be written as___

(a) $\log_c a$ (b) $\log_a c$

(c) $\log_a b$ (d) $\log_b c$

**9.** $\text{Log}_y x$ will be equal to___

(a) $\frac{\log_z x}{\log_y z}$ (b) $\frac{\log_x z}{\log_y z}$

(c) $\frac{\log_z x}{\log_z y}$ (d) $\frac{\log_z y}{\log_z x}$

**10.** For common logarithm, the base is_____

(a) 2 (b) 10

(c) e (d) 1

**11.** For natural logarithm, the base is____

(a) 10 (b) e

(c) 2 (d) 1

**12.** The integral part of the common logarithm of a number is called the____

(a) Characteristic (b) Mantissa

(c) Logarithm (d) None

**13.** The decimal part of the common logarithm of a number is called the _____:

(a) Characteristic (b) Mantissa

(c) Logarithm (d) None

14. If $x = \log y$, then y is called the ___ of x.
(a) Antilogarithm (b) Logarithm
(c) Characteristic (d) None

15. If the characteristic of the logarithm of a number is $\bar{2}$, that number will have zero (s) immediately after the decimal point.
(a) One (b) Two
(c) Three (d) Four

16. If the characteristic of the logarithm of a number is 1, that number will have ____ digits in its integral part
(a) 2
(b) 3
(c) 4
(d) 5

17. The value of x in $\log_3 x = 5$ is_____
(a) 243 (b) 143
(c) 200 (d) 144

18. The value of x in $\log x = 2.4543$ is
(a) 284.6 (b) 1.521
(c) 1.1010 (d) 0.4058

19. The number corresponding to a given logarithm is known as ___
(a) Logarithm (b) Antilogarithm
(c) Characteristic (d) None

20. 30600 in scientific notation is ___
(a) $3.06 \times 10^4$ (b) $3.006 \times 10^4$
(c) $30.6 \times 10^4$ (d) $306 \times 10^4$

21. $6.35 \times 10^6$ in ordinary notation is___
(a) 6350000 (b) 635000
(c) 6350 (d) 63500

22. A number written in the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer is called ____
(a) Scientific notation
(b) Ordinary notation
(c) Logarithm notation
(d) None

23. $\log p - \log q$ is same as
(a) $\log\left(\frac{q}{p}\right)$
(b) $\log(p-q)$
(c) $\frac{\log p}{\log q}$
(d) $\log\left(\frac{p}{q}\right)$



## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | c | 2. | b | 3. | d | 4. | a | 5. | b |
| 6. | a | 7. | c | 8. | a | 9. | c | 10. | b |
| 11. | b | 12. | a | 13. | b | 14. | a | 15. | a |
| 16. | a | 17. | a | 18. | a | 19. | b | 20. | a |
| 21. | a | 22. | a | 23. | d | | | | |